Candidate Name:

St Swithun's
WINCHESTER

Mathematics

Sixth Form Academic Assessment

Sample paper

Time allowed: 1 hour 30 minutes

Instructions to Candidates

All candidates should start at Question 1 and work through the paper until they finish or run out of time.

Each question is worth 5 marks but the intention is that questions increase in difficulty as the question number increases.

Please note that the diagrams given in these questions are not to scale.

You may use a calculator.
Do not write on the question paper. Write your answers on file paper.

Show all your working for each question.

Formulae

Sine Rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Rule:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Mathematics Sixth Form Academic Assessment

1. A pentagon has angles $2x$, $2x$, $2x$, $3x$ and $3x$.
What is the value of x?

2. Solve the inequality $5x - 7 \geq -2x + 8$

3. Make y the subject of the formula $x = \dfrac{3 - \sqrt{6-y}}{7ab}$

4. Without using a calculator, simplify $(\sqrt{27} - \sqrt{3})^2$

5. Solve:

   (a) $\frac{5}{x} = 35$

   (b) $5(2x + 7) - 3(4 - 8x) = 73$

   (c) $x^2 + 5x = 7x + 48$

6. Without using a calculator, find the exact value of

$$\frac{5\frac{1}{2} \times 1\frac{5}{33}}{4\frac{3}{4}}$$

7. Find the equation of the line that passes through the points $(-3, -1)$ and $(8, 2)$.

8. Harold has three sisters.
The mean age of the sisters is 27.
The mean age of Harold and his three sisters is 28. What is Harold's age?

9. The n^{th} term in a sequence is given by the formula

$$n^{\text{th}} \text{ term} = \frac{n}{2n-1}$$

Find the sum of the n^{th} term and the $(n + 2)^{\text{th}}$ term, giving your answer as a single fraction.

10. For what values of n does the equation $x^2 + nx - 24 = 0$ have integer solutions?

11. State the value of x for which y reaches its lowest possible value if

$$y = (x - 3)(x + 5) + 7.$$

Explain carefully how you arrive at your answer.

12. What is the largest possible radius of a circle that is tangent to both the x-axis and the y-axis, and passes through the point (5, 2)?

13. Find all positive integer solutions to the equation $5a + 3b = 19$, explaining your solution carefully.

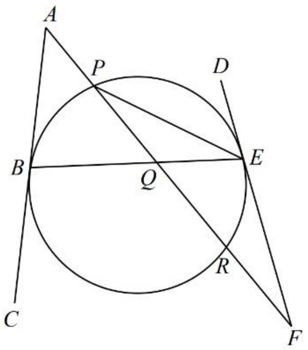
14. Factorise completely $x^4 - 1$.
Use this factorisation to write 1295 as a product of prime factors.

15. In the Fibonacci sequence 1, 1, 2, 3, 5, … , each term after the second is the sum of the previous two terms.
How many of the first 1000 terms of the sequence are odd?

16. Madge calculates the value of the expression $5^{1000} \times 8^{336}$.
How many digits does her answer contain? Justify your answer carefully.

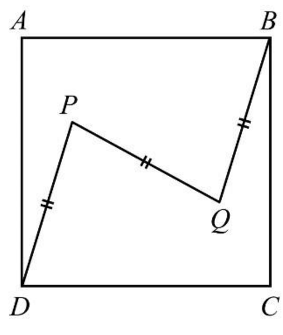
17. What is the largest two-digit number that is 75% greater when its digits are reversed?

18. In the diagram, *AC* and *DF* are tangents to the circle at *B* and *E*, respectively. Also, *AF* cuts the circle at *P* and *R*, and intersects *BE* at *Q*, as shown. If angle *CAF* is 35°, angle *DFA* is 30° and angle *FPE* is 25°, find the value of angle *PEQ*.



19. Prove that it is not possible to create a sequence of 4 numbers a, b, c, d, such that the sum of any two consecutive terms is positive, and the sum of any three consecutive terms is negative.

20. Points *P* and *Q* are inside square *ABCD* so that *DP* is parallel to *QB* and *DP* = *QB* = *PQ*. Determine the minimum possible value of angle *ADP*.



END OF QUESTIONS